



# Coupling BEM and GFPE for complex outdoor sound propagation

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In complex outdoor sound propagation meteorological effects as well as various absorbing properties and shapes of the boundaries need to be accounted for. This paper presents a hybrid GFPE-BEM method relying on the power of the BEM near obstacles and uneven topographies in order to compute the starting field which is then propagated thanks to the GFPE. The approach is firstly described. Then some numerical results are given for typical road traffic noise configurations with and without meteorological effects. The results show that this hybrid GFPE-BEM model can predict accurately the sound pressure levels in complex outdoor configurations.

## 1. INTRODUCTION

Due to the more and more demanding traffic noise regulations the sound pressure levels at receivers must be lower and lower. This implies that traffic noise has to be predicted at long ranges from the roads [1]. Consequently complex phenomena during sound propagation in an inhomogeneous atmosphere above uneven terrains with impedance discontinuities must be accounted for. Today there is a lack of models in order to meet this need. This paper aims at presenting a hybrid approach coupling two powerful numerical methods: the Boundary Element Method (BEM) [2] and the Green's Function Parabolic Equation (GFPE) [3]. The acoustic field next to obstacles like sound barriers, hills etc... is firstly computed with the BEM in a homogeneous atmosphere. Then this calculated field is used as the starter for the GFPE applied to the case of long range sound propagation above a flat ground with range-dependent meteorological profiles [4]. Numerical results are given for some typical road traffic noise configurations which show that this approach seems to be promising.

## 2. THEORETICAL APPROACH

### 2.1 The Boundary Element Method

This method that relies on the Integral Equation theory has been developed in the 60s and has been since extensively used [5]. Its main advantage is that it allows any kind of shape and absorption of the surfaces to be accounted for in a homogeneous atmosphere.

Here the BEM is based on a variational approach [2]. The geometry of the problem is bi-dimensional : the source is an infinite linear source and all the considered configurations remain unchanged and infinite along a direction perpendicular to the vertical section plane. The ground as well as any obstacle surface are reflective or can be characterized by their own acoustical admittance  $\beta$ . The theoretical formalism relies on an integral representation of the pressure field at any point as a function of the pressure on the boundaries, of the admittances

as well as of the Green's solution  $G$  (elementary solution for a point source  $M$  and for a receiver  $N$  above an absorbing ground) which can be written as the sum of three different terms

$$G(M, N) = -\frac{i}{4} H_0(kr) - \frac{i}{4} H_0(kr') + P_\beta(M, N) \quad (1)$$

where  $r$  is the distance between source and receiver,  $r'$  the distance between the image-source and the receiver and  $H_0$  the Hankel function of first kind and zero order. The second term in equation (1) represents the contribution of the reflection of the cylindrical wave on a perfectly rigid ground and the last term  $P_\beta$  is a corrective factor taking into account the ground admittance [6].

Although this method can be time consuming, it has proved to be very accurate for computing sound propagation in a homogeneous atmosphere above complex boundaries. The challenge is to adapt it to the case of sound propagation with meteorological effects. One way can be to include these meteorological effects in the Green's function [7]. In this paper an alternative approach is presented where the initial sound field is computed in a homogeneous atmosphere using the BEM and then propagated with the GFPE in a range-dependent inhomogeneous atmosphere.

## 2.2 The Green's Function Parabolic Equation

The parabolic approximation has been first introduced at the beginning of the 40s in order to solve electromagnetism problems. Later this theory has been applied to ocean acoustics and then to atmospheric sound propagation [8].

A known starting field is propagated step by step up to the receiver. Using the  $e^{-i\omega t}$  convention the Helmholtz equation can be rewritten as follows :

$$\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + k(r, z)^2 \right) P(r, z) = 0 \quad (2)$$

where  $k(r, z) = \frac{\omega}{c(r, z)}$  is the wavenumber and  $c(r, z)$  the sound speed.

Assuming that  $kr \gg 1$  and that there is a cylindrical symmetry, one can write for the solution of (2), with  $P(r, z) = \frac{1}{\sqrt{r}} u(r, z) e^{jk_0 r}$  :

$$u(r + \Delta r) = e^{j\Delta r \sqrt{Q}} u(r) \quad (3)$$

Here the backscattered field is neglected and the operator  $Q$  is defined by  $Q \approx \frac{\partial^2}{\partial z^2} + k^2$ .

After some developments [9], the acoustic pressure field can be finally given by :

$$u(r + \Delta r, z) = \frac{1}{2\pi} \left[ \int_{-\infty}^{+\infty} (U(r, k') + R(k')U(r - k')) \times e^{j\Delta r (\sqrt{k_r^2 - k'^2} - k_r)} e^{jk'z} dk' \right] + 2j\beta \times U(r, \beta) \times e^{j\Delta r (\sqrt{k_r^2 - \beta^2} - k_r)} e^{-j\beta z} \quad (4)$$

where  $U(r, k) = \int_0^{+\infty} e^{-jkz'} u(r, z') dz'$  is the Fourier transform of  $u(r, z)$  and  $\beta = \frac{k_r}{Z_g}$ .

The Parabolic Equation is a powerful method for describing sound propagation above a flat absorbing ground in an inhomogeneous medium. Impedance discontinuities can be included and range-dependent sound speed profiles due to temperature and wind profiles can be accounted for. Using the BEM around obstacles for computing the starting field allows an uneven topography to be described.

### 3. RESULTS

#### 3.1. Studied configurations

Two typical traffic noise configurations have been studied, including meteorological effects and an uneven topography.

The first case is a rigid T shape sound barrier [10] (Figure 1) the upper part of which is covered by a 5 cm thick layer of glasswool. The acoustic impedance of glasswool is characterized using Delany and Bazley's semi-empirical model [11] considering a flow resistivity  $\sigma_{glasswool} = 30 \text{ cgs}$ . The ground is rigid.

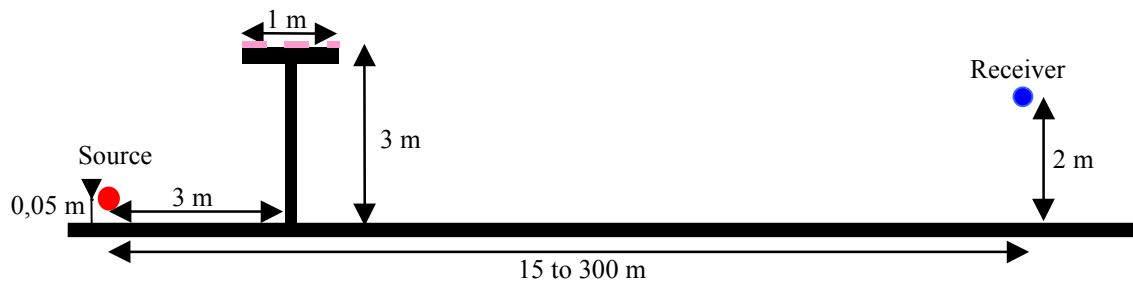


Figure 1. Geometry of the T shape noise barrier

The second studied case is a trench road in a countryside site. The surrounding ground is a grass-like surface of flow resistivity  $\sigma_{ground} = 300 \text{ cgs}$  (Figure 2). A 5 cm thick layer of glasswool covers the upper part of the trench on both sides. The bottom of the trench is rigid.

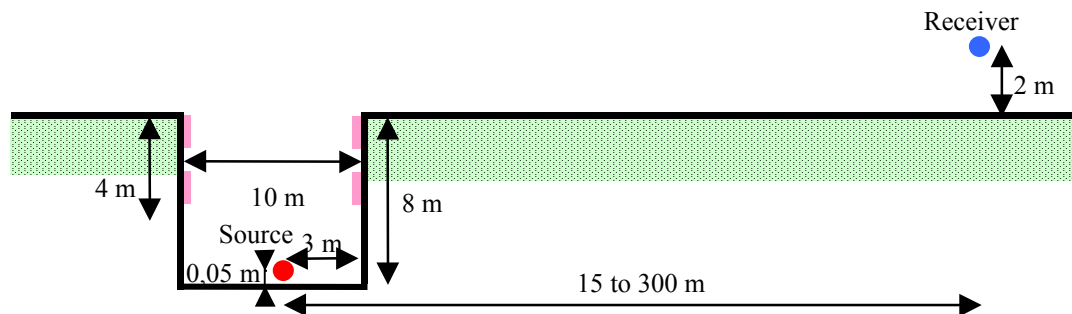


Figure 2. Geometry of the trench

For both studied configurations a point source is located at 5 cm above the ground. A set of receivers has been settled from 15 m up to 300 m far from the source at a height of 2 m above the ground.

### 3.2. Numerical results

Figures 4 and 5 display at a frequency of 1000 Hz the sound pressure levels relative to the free field as a function of the separation between source and receiver, respectively for the case of the T shape noise barrier (cf Figure 1) and for the case of the trench (cf Figure 2). The results are given for a homogeneous atmosphere and for an atmosphere with a logarithmic sound speed profile.

The acoustic pressure has been calculated in a homogeneous atmosphere with the BEM at 10 m far from the source. Then this sound field is used as the starter for the GFPE and propagated with this latter model in an inhomogeneous atmosphere up to the receiver (cf Figure 3). The meteorological effects are described by the following logarithmic sound speed profile :

$$c(z) = c_0 + \ln(1 + z/z_0) \text{ where } c_0 = 340 \text{ m.s}^{-1} \text{ and } z_0 = 0.1 \text{ m.}$$

Before computing the complex cases Figures 1 and 2 in a refracting atmosphere, the model has been validated in a homogeneous atmosphere. The results of this hybrid approach have been compared to reference results with the BEM.

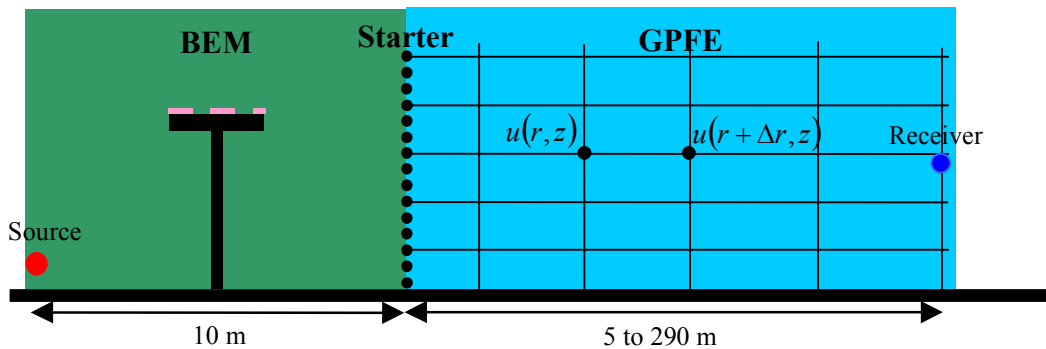
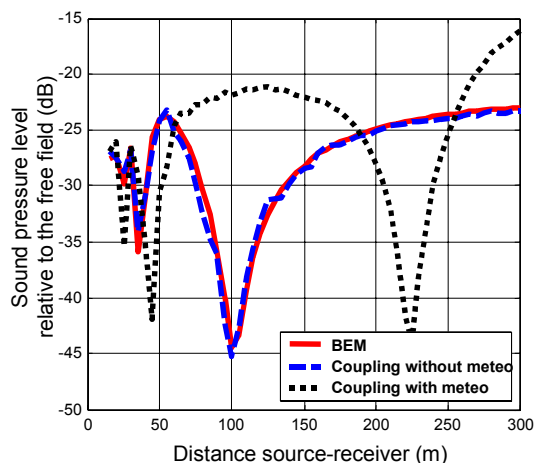
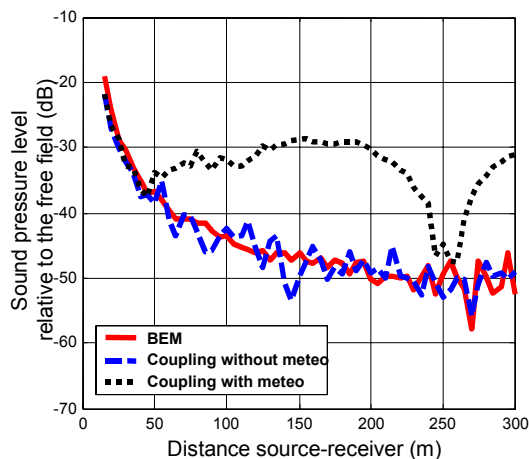


Figure 3. Sketch of the coupling



**Figure 4.** Sound pressure levels relative to the free field versus source-receiver distance. Frequency of 1000 Hz. Case of the T shape noise barrier (Figure 1).



**Figure 5.** Sound pressure levels relative to the free field versus source-receiver distance. Frequency of 1000 Hz. Case of the trench noise barrier (Figure 2).

A very good agreement between both numerical results has been found for both studied configurations (see figures 4 and 5). Then the cases of the T shape noise barrier and of the trench have been studied in the presence of a logarithmic sound speed profile. Figures 4 and 5 point out that the farther the receiver are located, the more important are the meteorological effects. Figures 4 and 5 show that the difference between the cases without and with meteorological effects can go up to 20 dB at around 200 m far from the source.

## 4. CONCLUSIONS

In this paper a hybrid BEM-GFPE method has been presented. In a homogeneous atmosphere the results have been compared satisfactorily to BEM calculations. This approach allows to describe complex sound propagation problems above uneven terrains in the presence of meteorological effects. Two typical traffic noise configurations have been studied pointing out the importance of meteorological effects.

## ACKNOWLEDGEMENTS

This work has been carried out within the frame of the European Project Harmonoise IST 2000-28419.

## REFERENCES

1. J. Defrance, Y. Gabillet, "A new analytical method for the calculation of outdoor noise propagation", *Applied acoustics* **57(2)**, pp.109-127, (1999).
2. P. Jean, J. Defrance and Y. Gabillet, "The importance of source type on the assessment of noise barriers", *JSV* **226(2)**, pp.201-216, (1999).



paper ID: 531-IP/p.6

3. N. Barrière, Y. Gabillet, “ Sound Propagation Over a Barrier with Realistic Wind Gradients. Comparison of Wind Tunnel Experiments with GFPE Computations, *Acustica - acta acustica* **85**, pp.325-334, (1999).
4. E. Premat, J. Defrance, M. Priour and F. Aballéa, “A hybrid GFPE-BEM approach for complex outdoor sound propagation”, in *Proceedings of LRSP 2002*, Grenoble (France), 2002.
5. R.D. Ciskowski, C.A. Brebbia, *Boundary Element Methods in Acoustics*, Elsevier Applied Science, London, 1991.
6. S. Chandler-Wilde, D.C. Hothersall, “Efficient calculation of the Green function for acoustic propagation above a homogeneous impedance plane”, *JSV* **185(5)**, pp.705-724, (1995).
7. E. Premat, Y. Gabillet, “A new boundary-element method for predicting outdoor sound propagation an application to the case of a sound barrier in the presence of downward refraction”, *J. A. S.A.* **108(6)**, pp.2775-2783, (2000).
8. K.E. Gilbert, M.J. White, “Application of the parabolic equation to sound propagation in a refracting atmosphere”, *J. A. S. A.* **85**, pp.630-637, (1989).
9. K.E. Gilbert, X. Di, “A fast Green’s function method for one-way sound propagation in the atmosphere”, *J. A. S. A.* **94(4)**, pp.2343-2352, (1993).
10. J. Defrance, P. Jean, and Y. Gabillet, “Acoustic performance of a T-shaped barrier cap”, in *Proceedings of Euro-Noise 98*, Munchen (Germany), 1998.
11. M.E. Delany and E.N. Bazley, “Acoustical properties of fibrous absorbent materials”, *Applied Acoustics* **3**, pp.105-116, (1970).